RL-augmented MPC for Humanoid Robot Locomotion

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INTRODUCTION

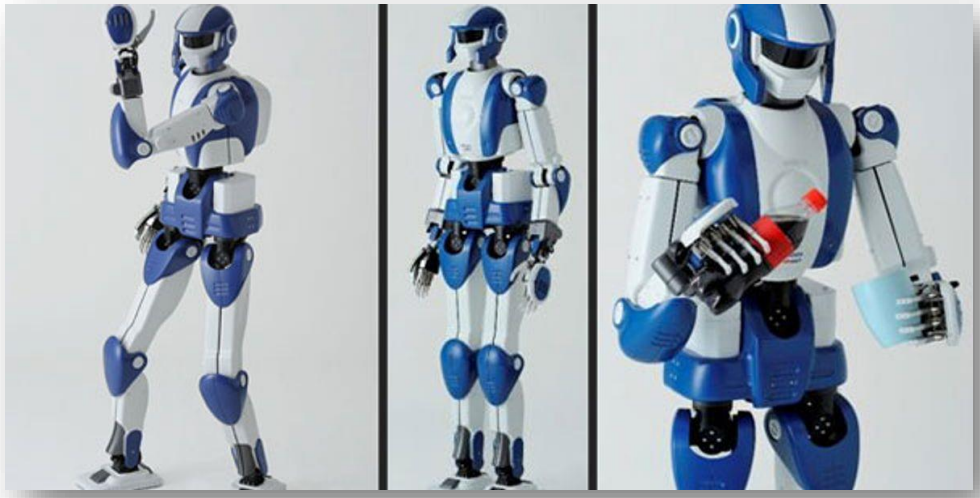
Introduction

Humanoid robots

- More and more ubiquitous
- Many DOFs, underactuated
- Models are complex, need simplifying assumptions

In this project:

- Walking is based on the LIP dynamics: control CoM-ZMP system, footsteps as constraints
- We add a RL-trained agent to modify footsteps to react to external disturbance forces and terrain inclination



Introduction

LIP Dynamics

- Linear model obtained assuming that
  - Center of mass is at constant height: $z_c = h$
  - Internal angular momentum is constant: $\dot{L}_c = 0$

- Uses ZMP to control CoM + dynamic extension

with $\eta = \sqrt{\frac{g}{h}}$

$$\frac{d}{dt}\begin{bmatrix} p_c \\ \dot{p}_c \\ p_z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \eta^2 & 0 & -\eta^2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} p_c \\ \dot{p}_c \\ p_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\dot{p}_z$$

Introduction

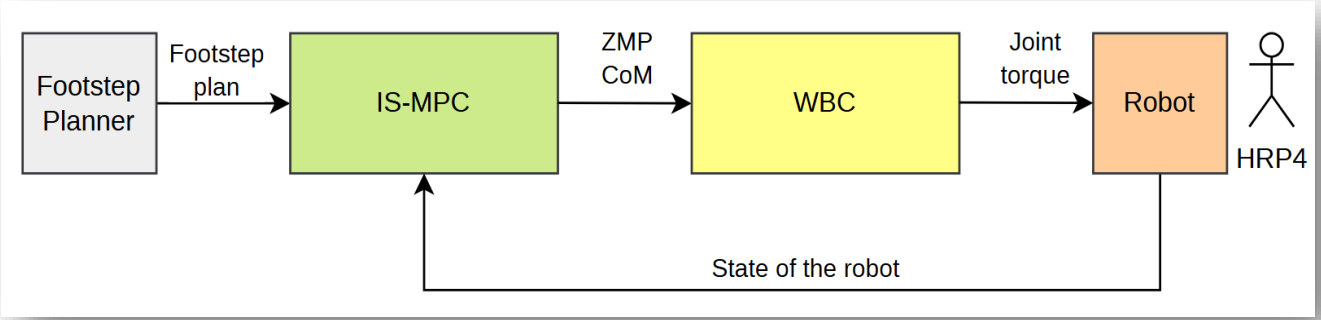
IS-MPC

- Extension to classical MPC that includes **stability contraints**
- Uses LIP model for prediction
  - Uses ZMP to control CoM
  - CoM position and velocity and ZMP position as state
  - ZMP velocity as control input (dynamic extension)

- Footsteps are used to generate constraints for support polygon
- Formulation includes constraints on **boundedness of CoM trajectory** w.r.t ZMP trajectory
  - If these constraints are satisfied , the dynamics is stable

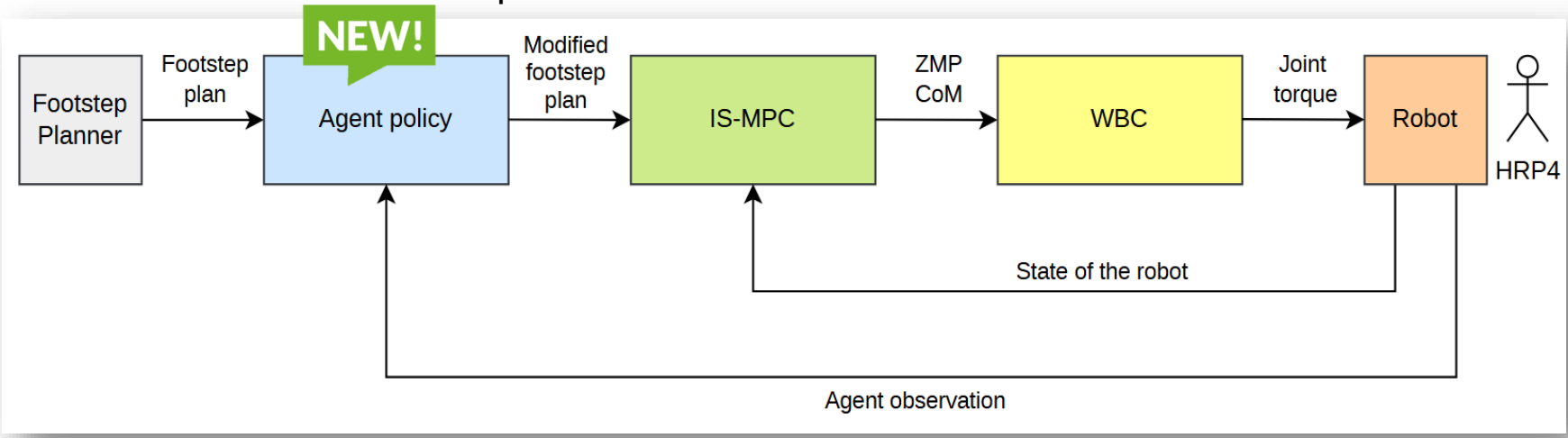
- Fixed timings for single and double support phases
  - Reference paper considers them variable

Introduction



Reinforcement Learning approach

- IS-MPC prediction model is not accurate in the presence of high disturbance forces and terrain inclination
- Augment the scheme with a RL-trained agent which displaces the footsteps to make the robot keep balance



RL approach

PPO – On Policy Algorithms

1) **Act** according to the current policy while collecting data (Roll-out)
2) **Process** data to compute the **Advantage** (used in loss function)
3) **Update** the policy network w.r.t. PPO **Loss** to maximize the **Expected Return**
4) Repeat

- ❑ When enough data is collected the simulations are paused and network is updated
- ❑ **Advantage** is used to compute part of the PPO Surrogate Loss
- ❑ Then Roll-outs are restarted and simulations are resumed
- ❑ This process is repeated for a large number of episodes
- ❑ Our implementation runs **16 parallelized environments** each Roll-out

Actor-Critic algorithm

The **advantage** function express how good it is to take a certain action *a* in state *s* w.r.t. the other actions

It is defined as:

Estimated by a NN

$$A(s,a) = Q(s,a) - V(s)$$

Expected return from taking action *a* in state *s*

Mean expected return of the policy in state *s*

- It is used to compute an **estimator of the policy gradient**
- **Gradient ascent algorithm** to update the policy network (**actor** component)
- **V(s)** estimated by the **critic** **component** of the neural policy
- **V(s)** estimation updated according to a squared-error loss w.r.t. a **V(s)** target

RL Curriculum Learning

Curriculum Learning

- Start by training the agent on simple environments
- **Gradually increase the difficulty** of the environment in **levels (**not included in reward function)
- Difficulty corresponds to range of possible terrain inclinations
  - Range [3.4°, 6.8°] * level / 100

- An agent starts from 0 points.
  - ✓ It is awarded 2 points for successfully completing a foostep plan with no failure.
  - ✓ 1 point subtracted on any failure
- An agent **levels up** when it accumulates a total of 6 points

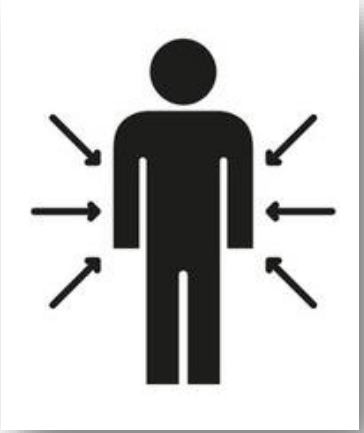
Disturbances

External forces

- Random external forces between 30 and 87N are applied to random bodies of the robot (torso, body, feet)

Ground Slope

- The ground has a slope (both in x and y direction) that is not modelled in MPC



GENERAL FRAMEWORK

Observations

Observations

$$\mathcal{S} = \begin{bmatrix} \sigma_{\mathbf{k}} & \Delta t_{r,k} & \mathbf{p_{c,k}} & \dot{\mathbf{p}}_{\mathbf{c,k}} & \mathbf{p_{z,k}} & \dot{\mathbf{p}}^{\mathbf{r}}_{\mathbf{c,k}} & \mathbf{p}^{\mathbf{r}}_{\mathbf{z,k}} & \mathbf{L} & \boldsymbol{\Delta}\sigma_{\mathbf{k+1}} & \boldsymbol{\Delta}\sigma^{\mathbf{r}}_{\mathbf{k+1}} \end{bmatrix}^T \in \mathbb{R}^{25} \times \mathbb{SO}^2(2)$$

- $\sigma_{\mathbf{k}}$ Support foot encoding
- $\Delta t_{r,k}$ Remaining time in swing
- $\mathbf{p_{c,k}}$ $\dot{\mathbf{p}}_{\mathbf{c,k}}$ CoM position and velocity
- $\mathbf{p_{z,k}}$ ZMP position
- $\dot{\mathbf{p}}^{\mathbf{r}}_{\mathbf{c,k}}$ CoM reference velocity
- $\mathbf{p}^{\mathbf{r}}_{\mathbf{z,k}}$ ZMP reference position
- $\mathbf{L}$ Angular momentum
- $\boldsymbol{\Delta}\sigma_{\mathbf{k+1}}$ Next footstep position
- $\boldsymbol{\Delta}\sigma^{\mathbf{r}}_{\mathbf{k+1}}$ Next footstep reference position

All components of the state are relative to the current support foot

Displacements given in this form

$$\Delta\sigma_{k+1} = \begin{bmatrix} \Delta x & \Delta y & \Delta\varphi \end{bmatrix}^T \in \mathbb{R}^2 \times \mathbb{SO}(2)$$

Rewards

Reward function given to the Agent

$$R(s_k, a_k, s_{k+1}) = r_{\text{state}} + r_{\text{phase}} + r_{\text{cp}} + r_{\text{CoM}} + r_{\text{fs}}$$

$$r_{\text{state}} = \begin{cases} \text{alive bonus} & \text{if robot alive} \\ \text{penalty} & \text{if IS-MPC solver solution innacurate} \\ \text{penalty} \cdot 0.5 & \text{if IS-MPC solver max iters.} \\ \text{penalty} \cdot 3 & \text{if Inverse Dynamics not feasible} \\ \text{penalty} \cdot 5 & \text{if feet collide} \end{cases}$$

$$r_{\text{phase}} = \begin{cases} r_{\text{swing}} & \text{if is in swing phase} \\ r_{\text{ds}} & \text{if is in double support} \end{cases}$$

$$r_{\text{swing}} = -w_h(z_c - h)^2 + w_a \frac{a^T a}{\Delta t_r + 10^{-3}}$$

$$r_{ds} = -w_a a^T a$$

$$r_{\text{cp}} = \begin{cases} \text{checkpoint bonus} & \text{if robot reaches multiple of 3 footsteps} \\ \text{checkpoint + end bonus} & \text{if robot reaches the end of the plan} \\ 0 & \text{otherwise} \end{cases}$$

$$r_{fs} = r_{fs_1} + r_{fs_2}$$
$$r_{fs_1} = -\text{Ker}_f(\Delta f^2)$$
$$r_{fs_2} = \text{Ker}_f(|\Delta f^r|^2)$$

$$r_{CoM} = \text{Ker}_{\dot{x}_c}(e_{\dot{x}_c, \dot{x}_c^r}) + \text{Ker}_{\dot{y}_c}(e_{\dot{y}_c, \dot{y}_c^r})$$

$$\text{Ker}_v(e) = w_v \exp(-(e/\sigma_v)^2)$$

Different Approaches

Different approaches to footstep displacement

- Approach 1: displace only the next footstep.
- Approach 2: displace the whole plan.
- Approach 3: fully displace the next footstep, then geometrically scale displacement of successive footsteps.

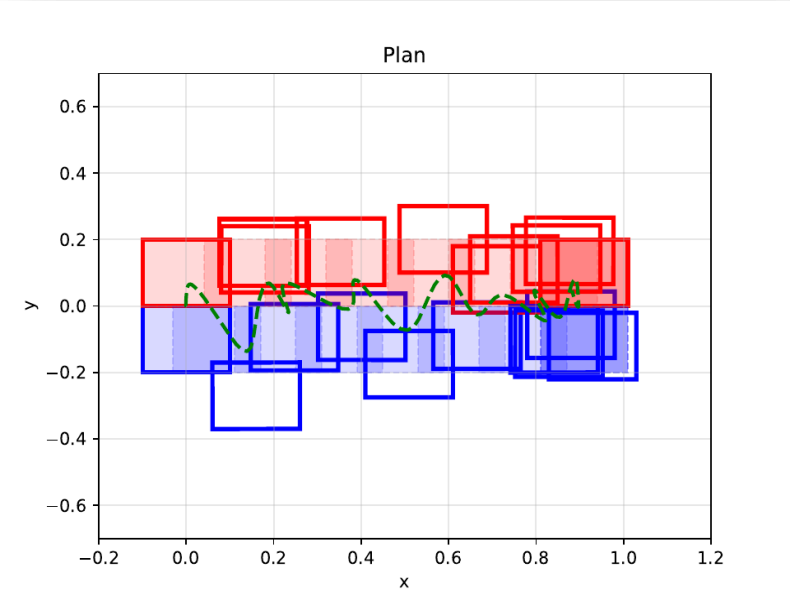
- Saturations are considered on the maximum displacement of the next footstep to ensure it stays feasible after displacement (for all agents).

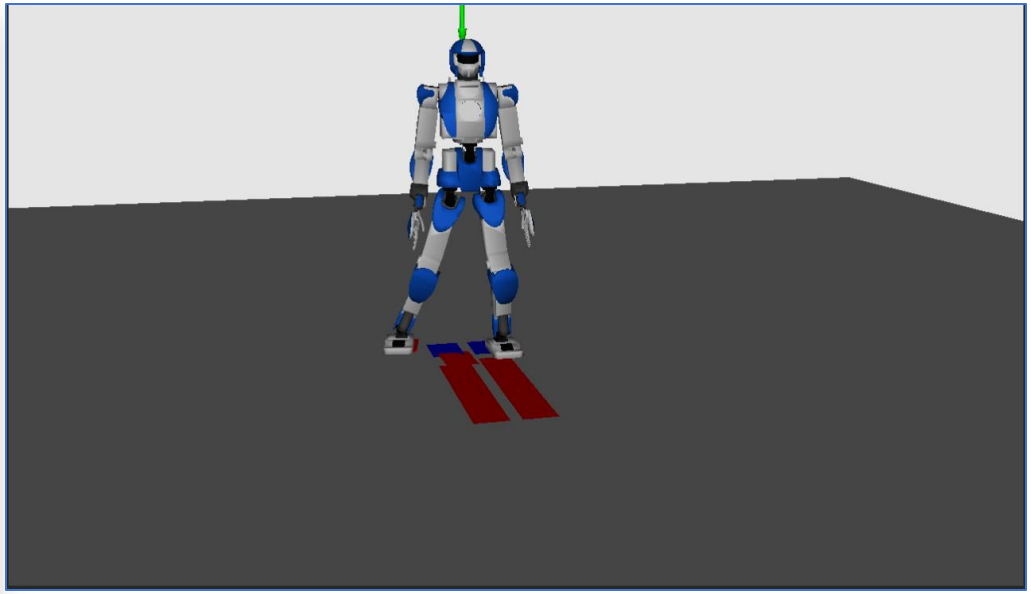
- Since in PPO actions are sampled from a Gaussian policy, in general we expect to see some displacement even when no disturbance is present, even on a very well trained agent.

Footstep displacement

Approach 1

- Only the next foostep is displaced
- Little displacement, easily track the nominal footstep plan
- Displacement might make the plan unfeasible, especially when caused by external forces or terrain inclination
  - Feasibility of current displacement depends on future (unknown) actions!

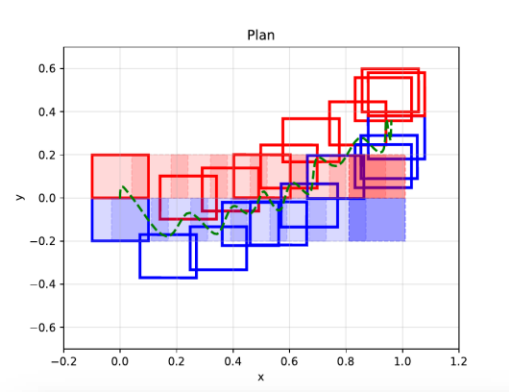


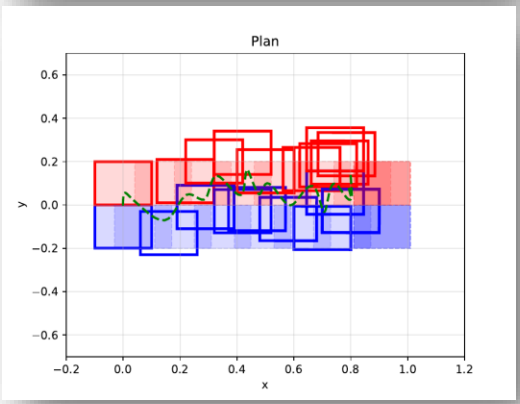


Footstep displacement

Approach 2

- The whole plan is displaced
- Displaced plan is feasible if both the original plan and the current displacement are
- Displacements accumulate, the robot easily strays from the nominal plan
  - Can add appropriate reward to encourage better tracking, but this doesn't always work



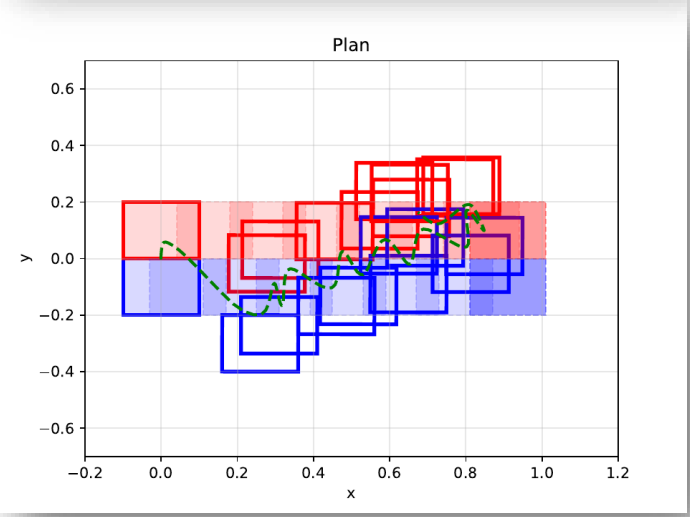


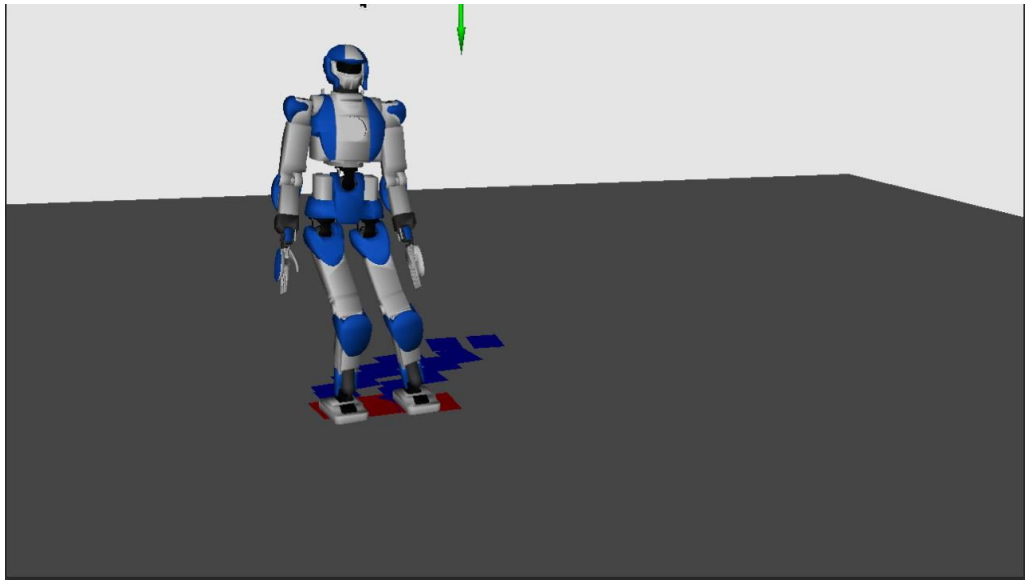
Footstep displacement

Approach 3

- Tradeoff between approaches 1 and 2
- The first footstep is completely displaced, for the successive ones the displacement is geometrically scaled by a fixed factor (hyperparameter)
- Better tracking that approach 2, displacements are feasible with higher probability than approach 1.

$$FS'_{t+i} = FS_{t+i} + \xi^{i-1}\Delta_t, \; i = 1, 2, ..., \Sigma$$





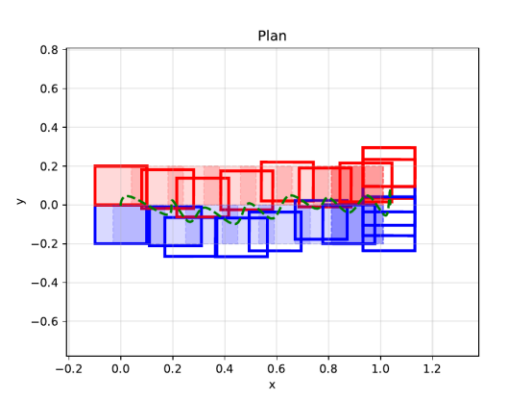
Footstep displacement

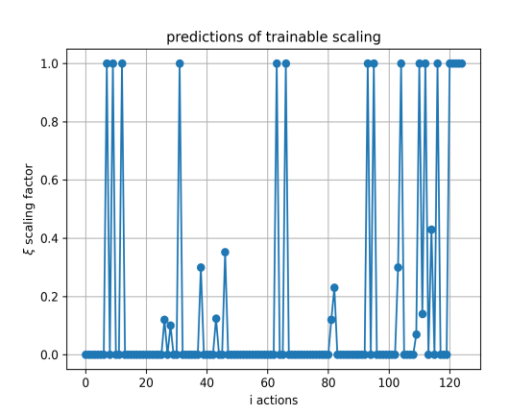
Approach 3- Extension

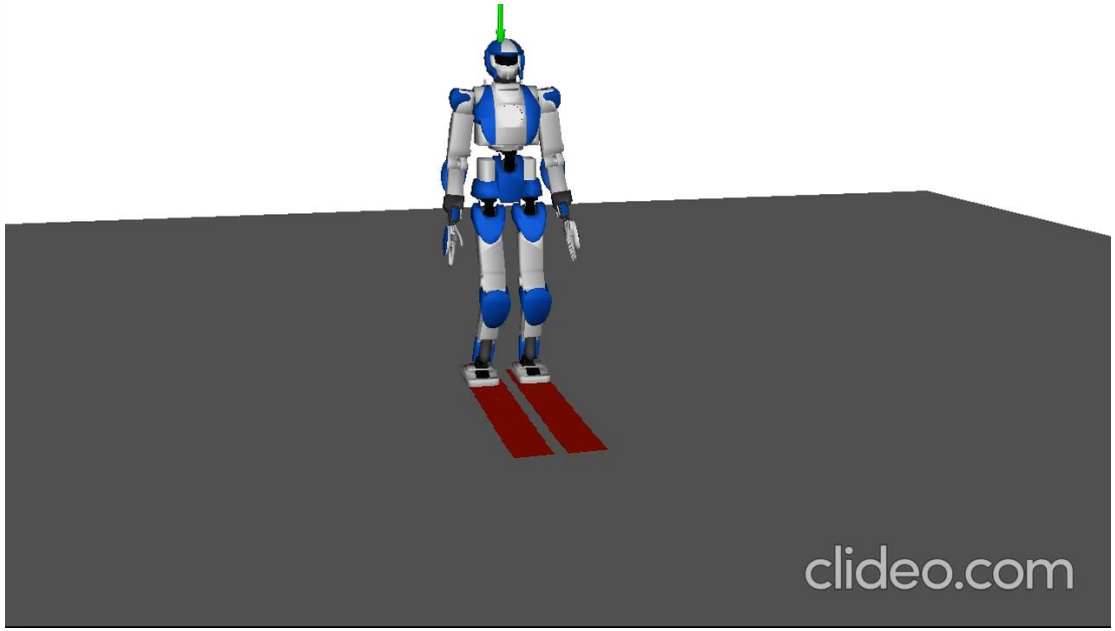
Approach 3 with the scaling parameter that is **learned.**

$$A_{int} = \{\Delta_x, \Delta_y, \Delta_\theta\}$$

$$A_p = \{\xi\}$$

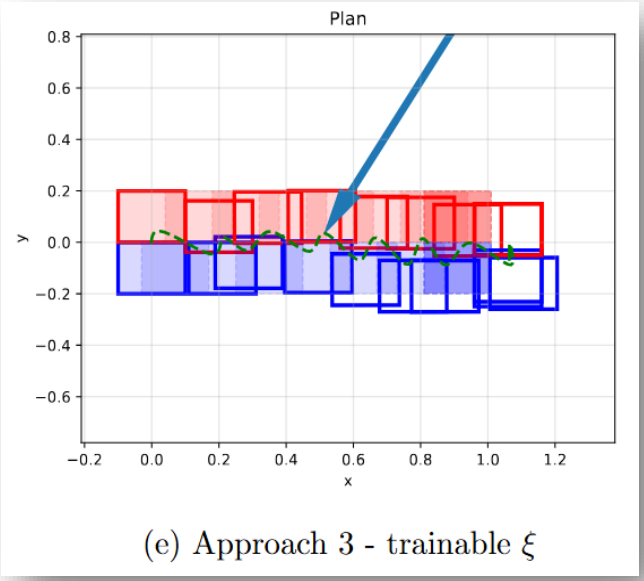






Nominal Conditions: displace only 1st footstep

Foostep displacement

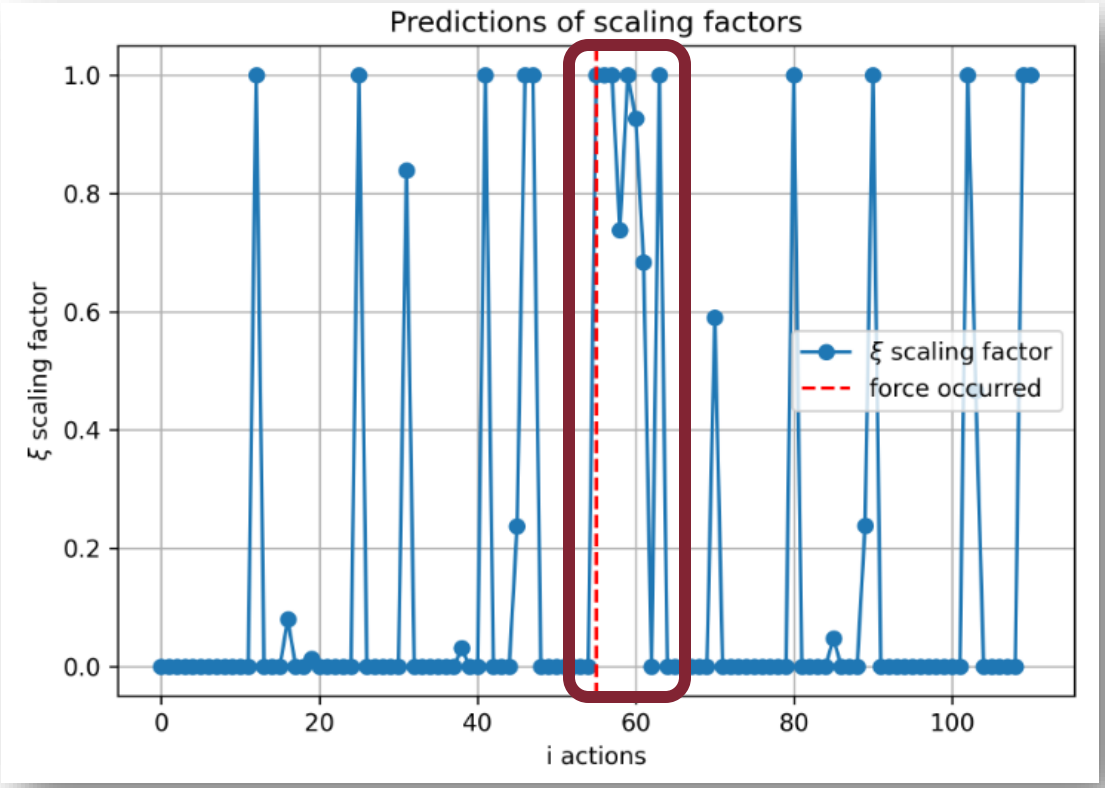


(e) Approach 3 - trainable ξ

More uncertain behavior if **ground slope** is introduced

Only external forces

Learned to modify all the plan if it is needed to recover **dynamical balance** after **force disturbances**



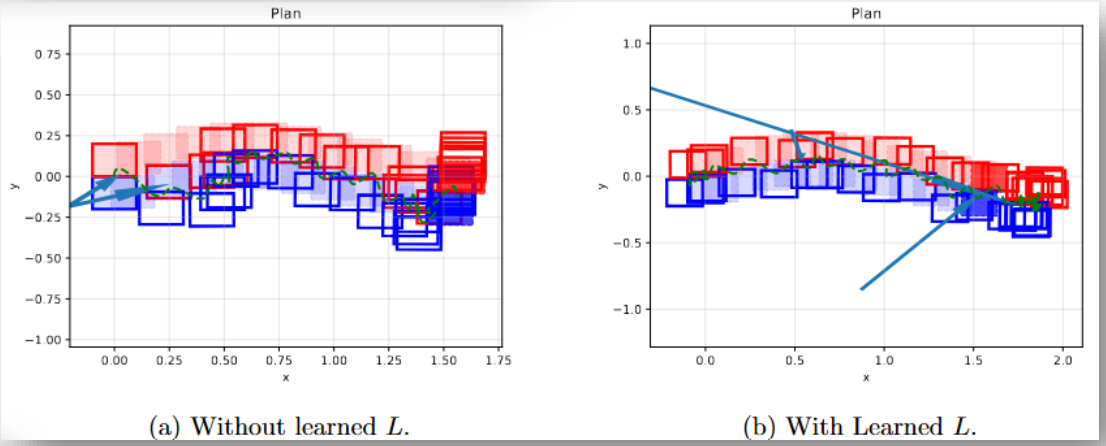
Learning Angular Momentum

Idea: Learning to reproduce a Desired Angular Momentum about the contact point

We get an Angular Momentum reference performing a simulation in **nominal conditions** with **only IS-MPC**

$$L = [L_x, L_y] \qquad \mathbf{L_{des}} = [L_{xd}, L_{yd}] \qquad \mathbf{e_L} = \mathbf{L_{des}} - \mathbf{L}$$

augmented agent observation space



(a) Without learned L. (b) With Learned L.

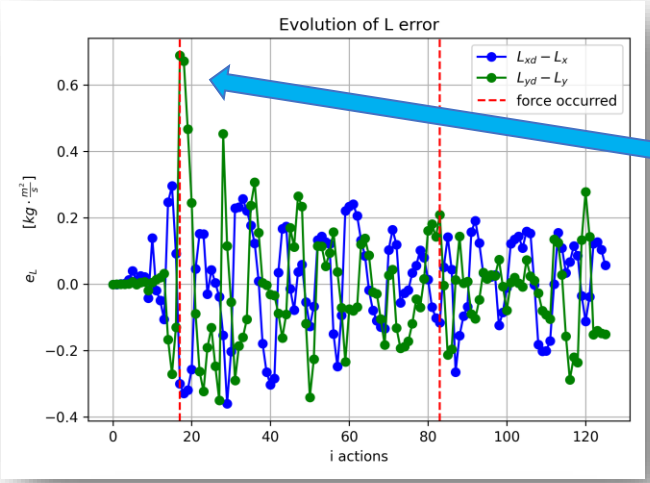
Promoting tracking with the **reward term:**

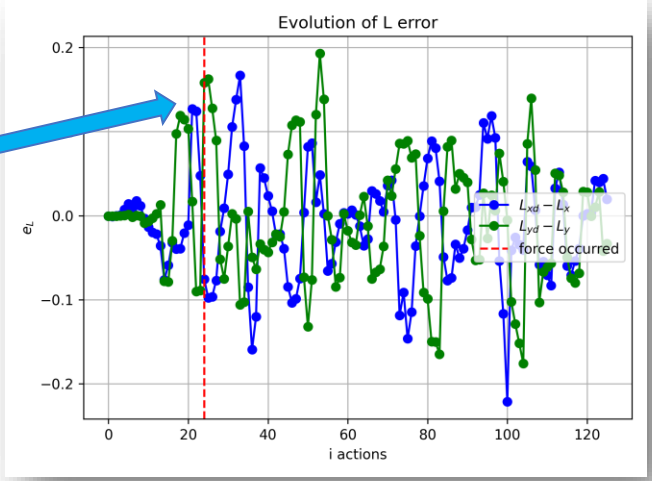
$$r_{L_{des}} = Ker_L(|\mathbf{e_L}|)$$

better **reference reproduction** capabilites

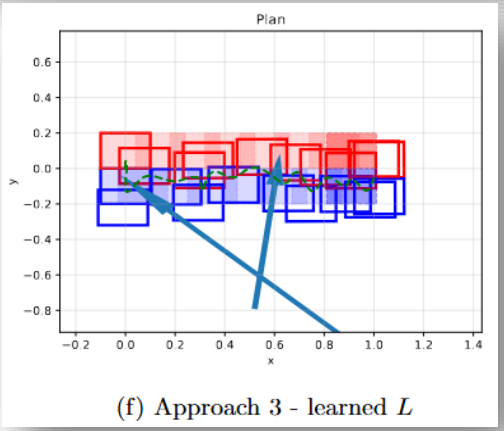
Learning Angular Momentum

Idea: Learning to reproduce a Desired Angular Momentum about the contact point



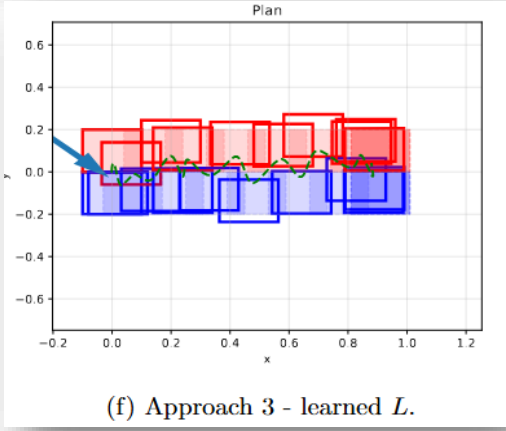


Spikes in the error as an **external force** occurs



(f) Approach 3 - learned L

not so effective in presence of **ground slope**



(f) Approach 3 - learned L.

SIMULATIONS

Simulations

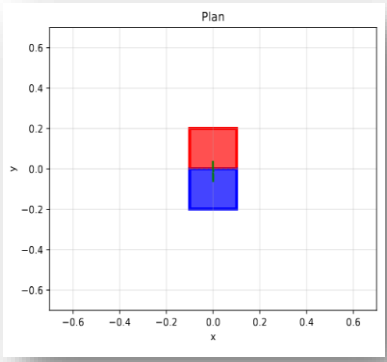
Setup

- **Sampling time** 0.01s
- IS-MPC **prediction horizon** to 150 samples (1.5s)
- Policy is a **MLP**, two hidden layers of 64 nodes each and *tanh* activations
- Saturation on total displacement of the next foostep is set to 0.2m
- Only approach 3 is shown for brevity, scaling factor set to 0.9
  - The report includes each simulation with all three approaches
- Random **forces between [30, 87] [N]** in magnitude, applied with 3% chance each step to a random part of the robot between torso, body and feet
- **Feet collision** is considered a failure, same as crashing MPC or Whole Body Controller solvers

Simulations

Sim 2 – **Walking on the spot**, with forces, no terrain inclination

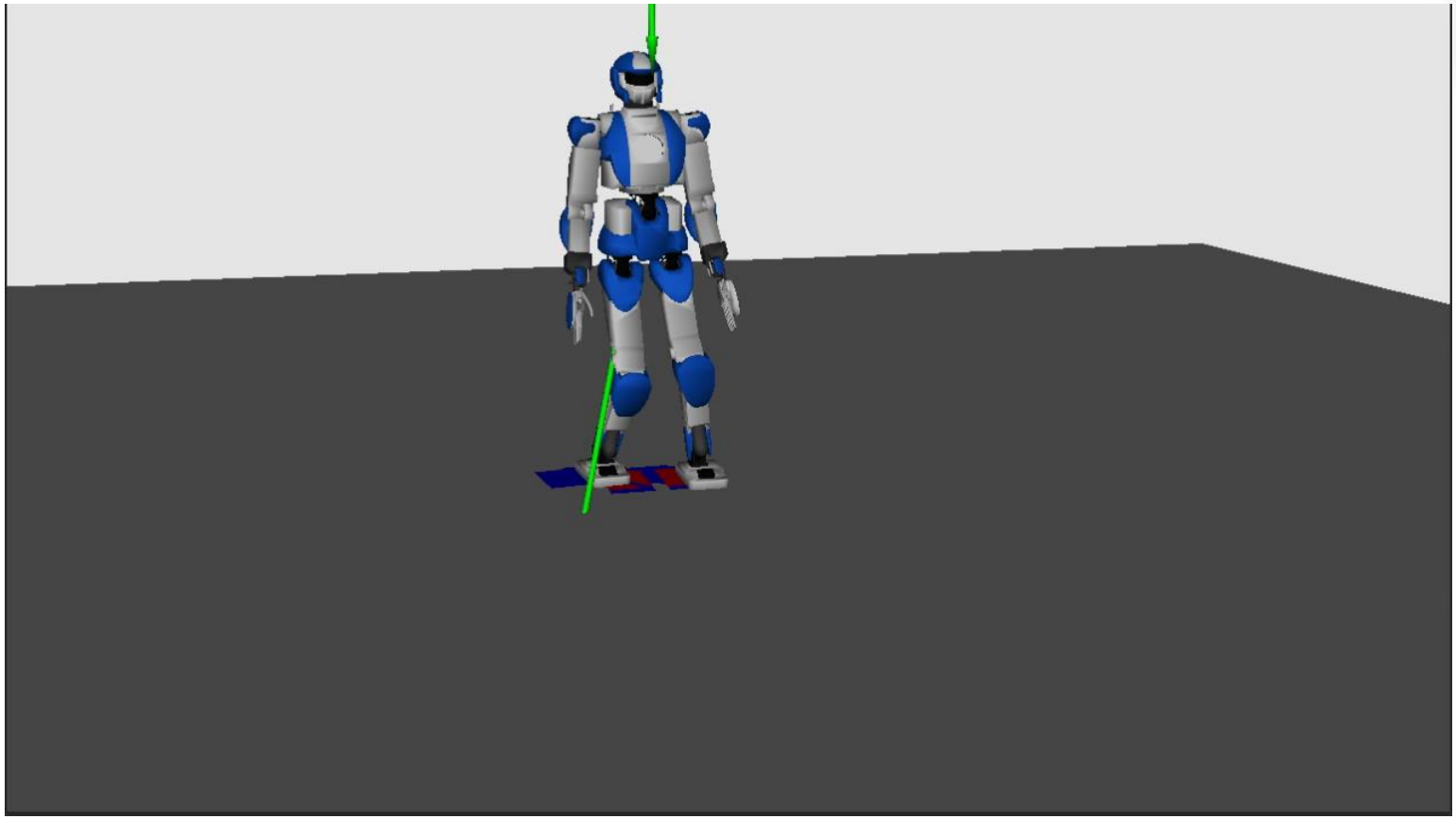
- Nominal case (IS-MPC only)

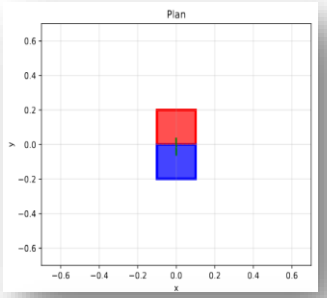


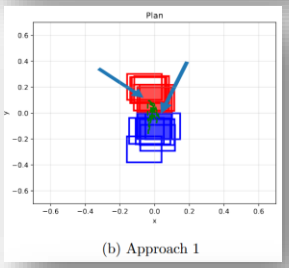
Simulations

Sim 2 – **Walking on the spot**, with forces, no terrain inclination

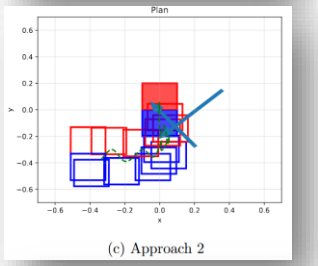
- Approach 3



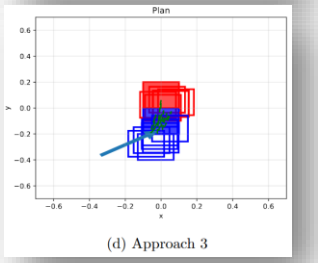




(b) Approach 1



(c) Approach 2

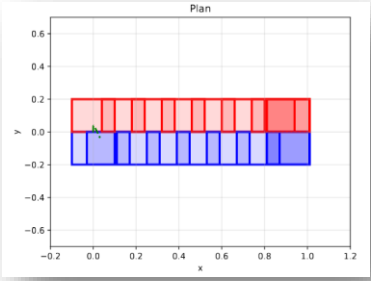


(d) Approach 3

Simulations

Sim 3 – **Straight line trajectory**, with forces and inclination (x: 3° , y: 1°)

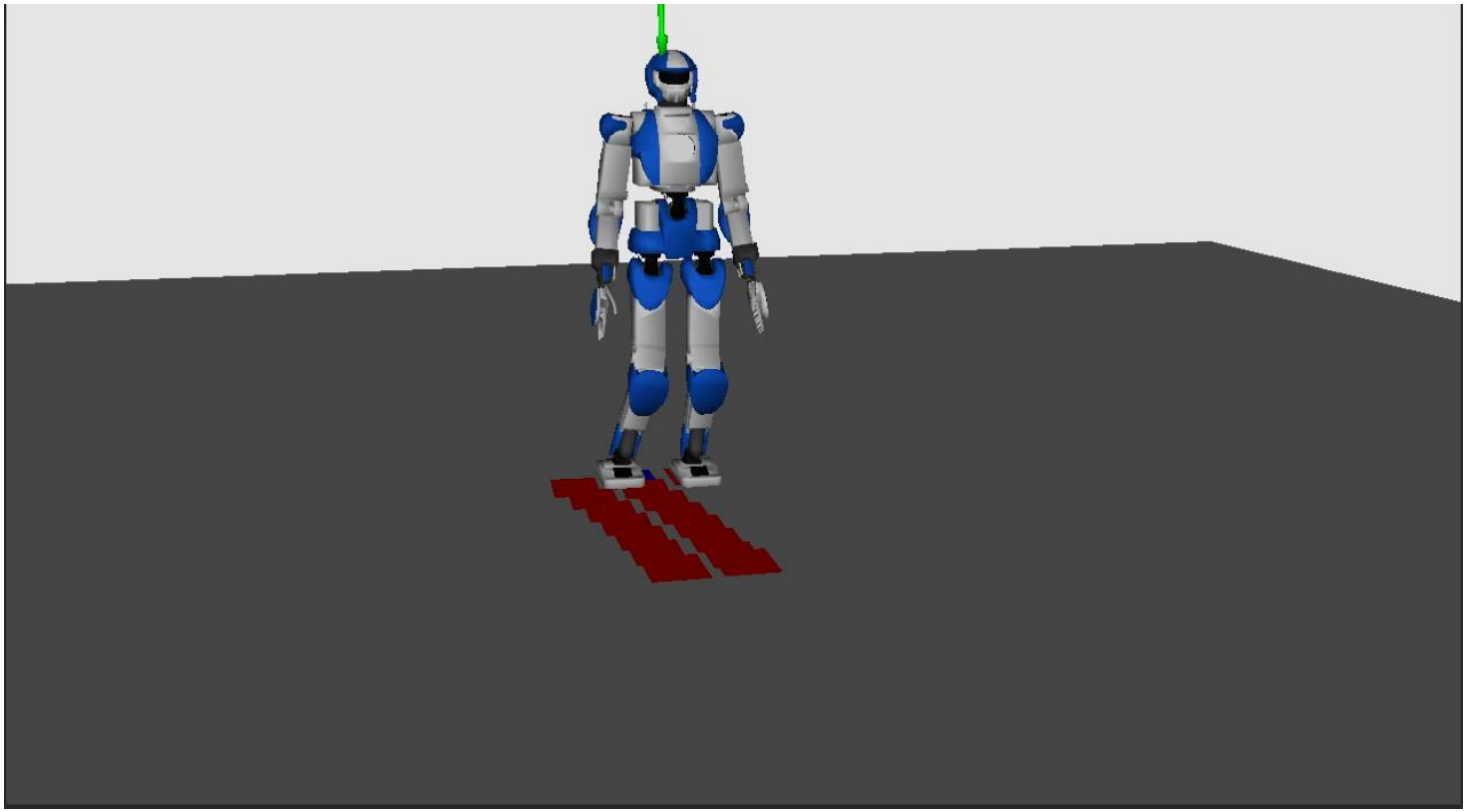
- Nominal (IS-MPC only)

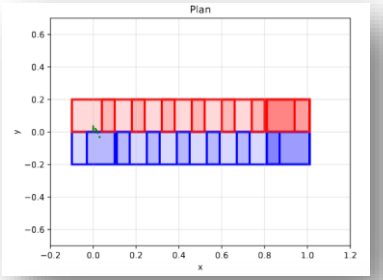


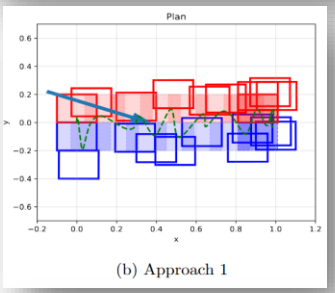
Simulations

Sim 3 – **Straight line trajectory**, with forces and inclination (x: 3° , y: 1°

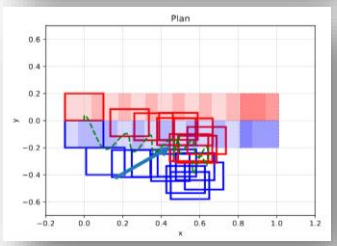
- Approach 3

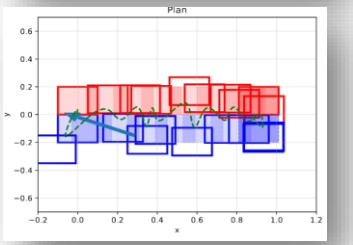






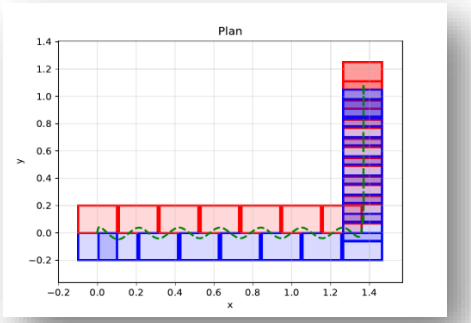
(b) Approach 1



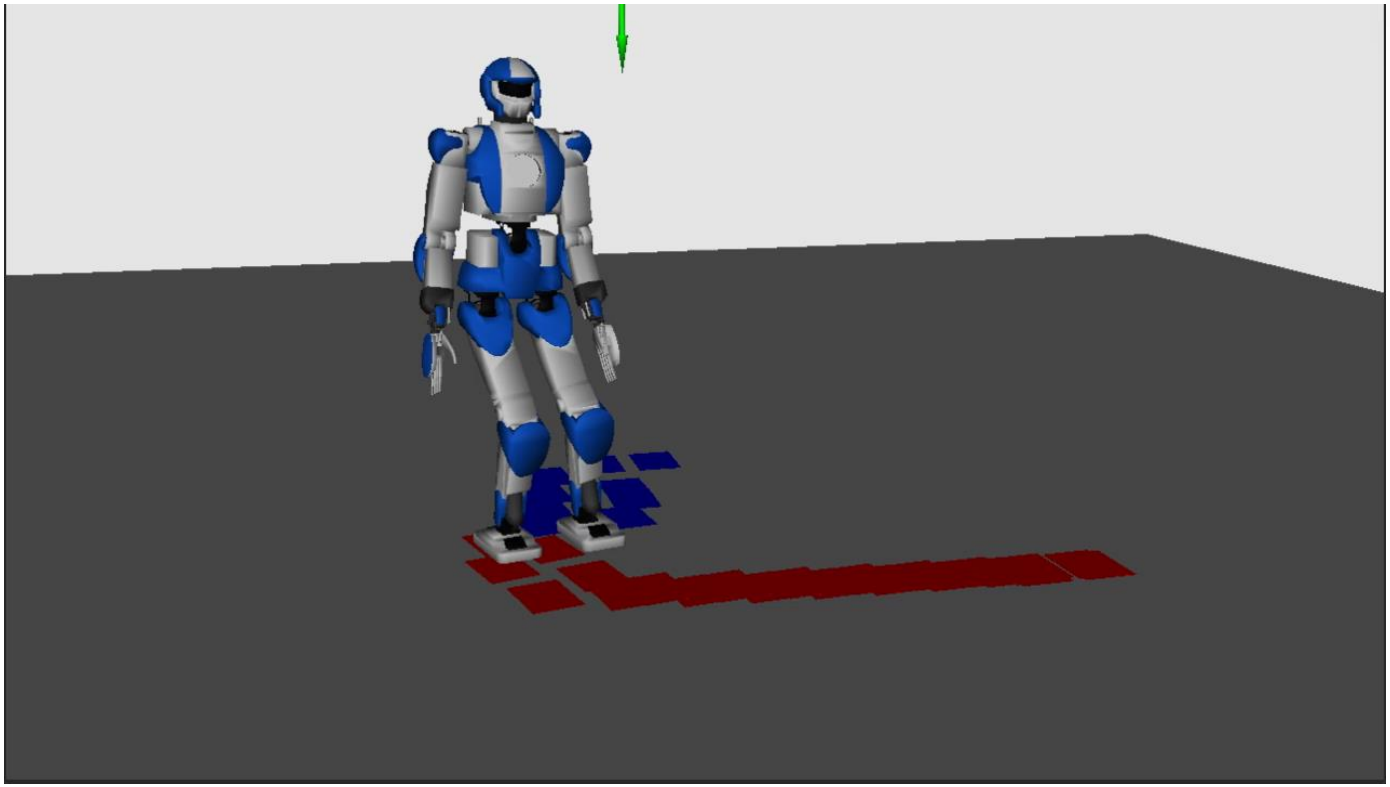


Simulations

Sim 4 – **Validation trajectory**, with forces, no inclination



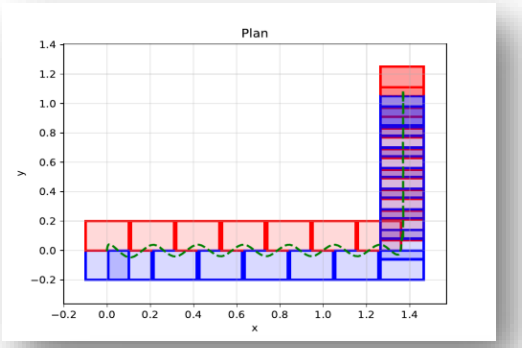
- Approach 3
- Most common modes of failure are feet collision or solver failure when changing direction: the agent is **not generalizing** well

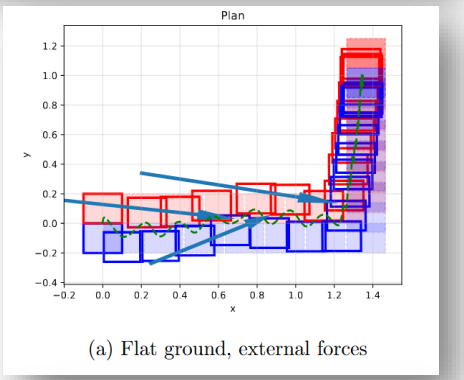


Simulations

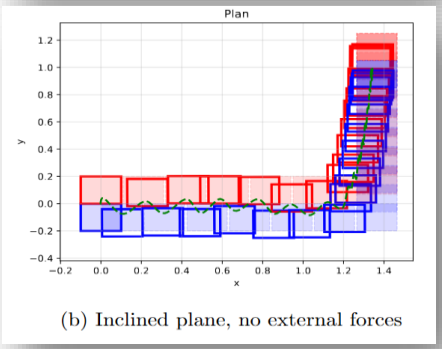
Sim 5 – **Validation trajectory**, with forces, inclination (x: 3° , y: 1°) and saturated actions

- Approach 3 – Trainable scaling factor and learned L
- Actions are saturated to prevent feet collision and for max. displacement, model trained from scratch





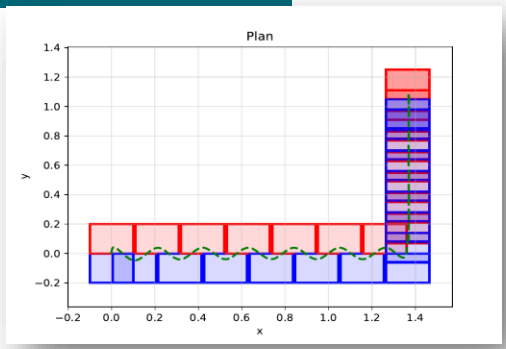
(a) Flat ground, external forces



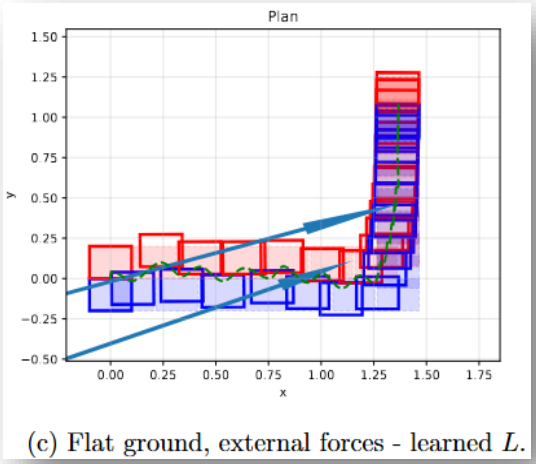
(b) Inclined plane, no external forces

Simulations

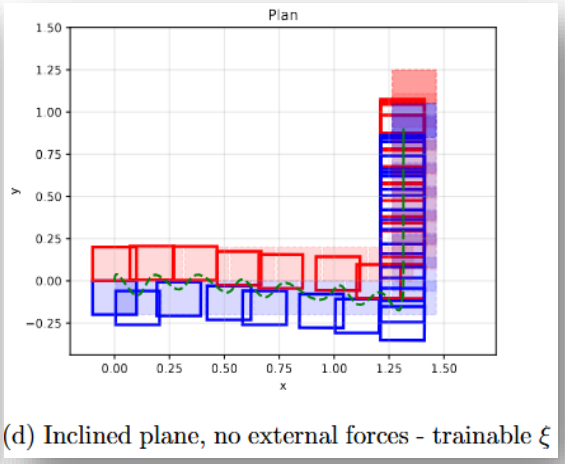
Sim 5 – **Validation trajectory**, with forces, inclination (x: 4° , y: 1°) and saturated actions



- Approach 3, trainable scaling factor and learned L

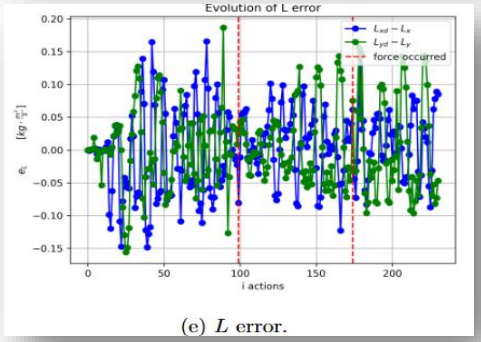


(c) Flat ground, external forces - learned L.

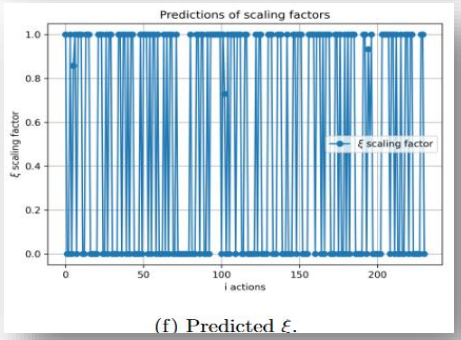


(d) Inclined plane, no external forces - trainable ξ

actions saturation allows to walk sideways



(e) L error.



(f) Predicted ξ.

more involved scaling factor predictions if **ground** is **tilted**

When To Use The Agent?

Use the agent **only** when is needed

- The agent will always modify the plan
  - But if well trained, the displacements will be close to zero when in near-nominal conditions
- It is interesting to find a way to "turn-off" and "turn-on" the agent only when a force is applied or the terrain is inclined.
- How to detect if there are perturbations in the environment?

Energy Observer

Hybrid Gaussian-Bernoullian decision policy

Energy Observer Derivation

Energy observer for **perturbation detection**

- The energy of the LIP dynamics can be monitored with a **residual**
- If the residual is above a threshold, something is perturbing the model and the Agent must take actions

$$E(t) = K(t) = \frac{1}{2\eta^2}\dot{p}^T\dot{p}$$

$$\dot{E}(t) = \frac{1}{\eta^2}\dot{p}_c^T\ddot{p}_c = \dot{p}_c^T(p_c - p_z) - \frac{g}{\eta^2}y_c$$

$$\ddot{p}_c = \begin{bmatrix} \ddot{x}_c \\ \ddot{y}_x \\ \ddot{z}_c \end{bmatrix} = \begin{bmatrix} \eta^2(x_c - x_z) \\ \eta^2(y_c - y_z) \\ \eta^2(z_c - z_z) - g \end{bmatrix}$$

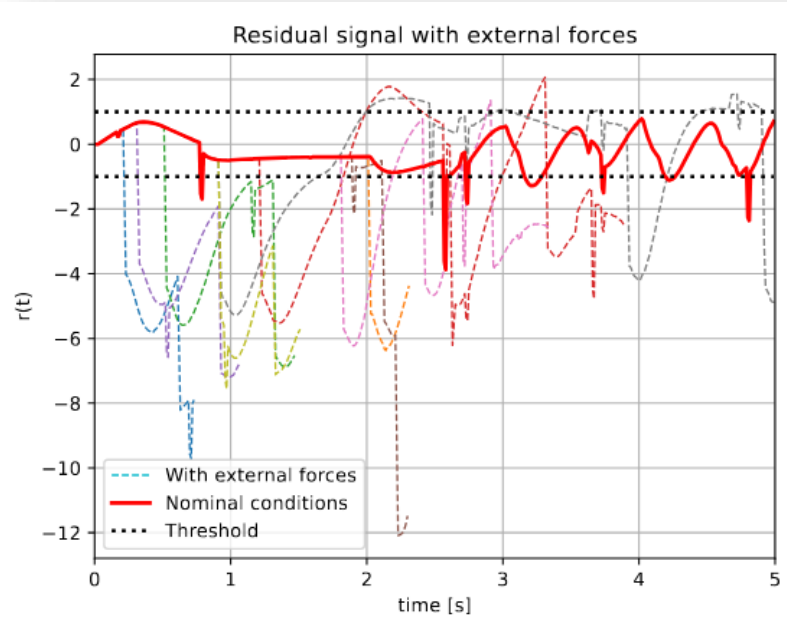
$$\dot{r}(t) = G\left(\dot{E}(t) - \dot{p}_c^T(p_c - p_z) + \frac{g}{\eta^2}y_c - r(t)\right) = G\Big(n(t) - r(t)\Big)$$

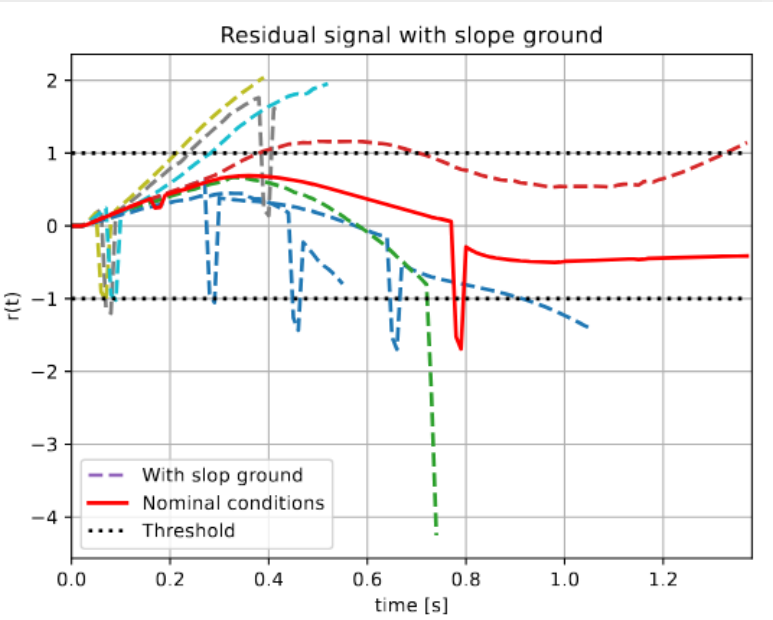
$$\mathcal{L}\{\dot{r}(t)\} = sr(s) = G(n(s) - r(s)) \Rightarrow r(s) = \frac{G}{G+s}n(s)$$

Energy Observer

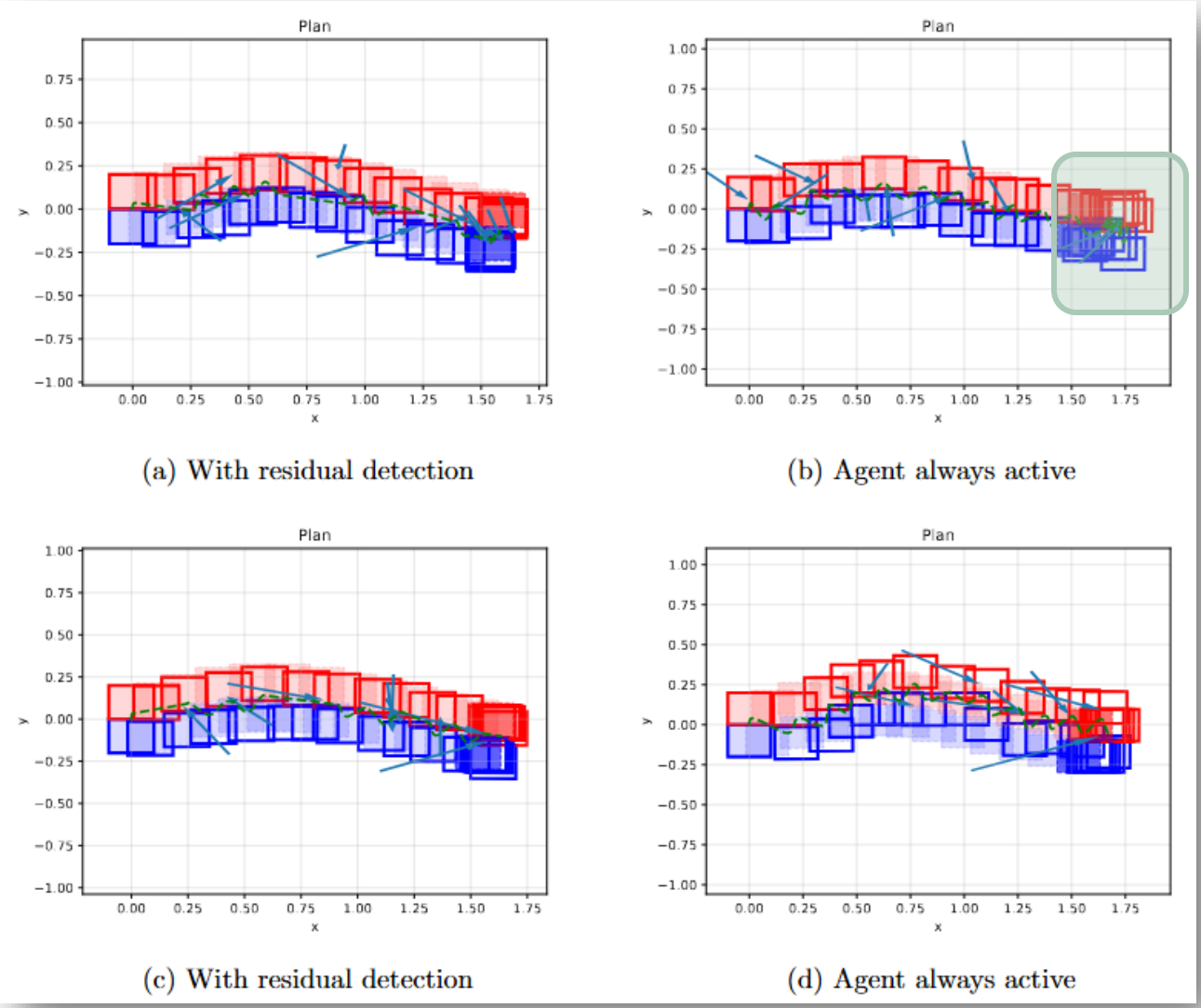
Energy observer for **perturbation detection**

$$r(t) = G\left(E(t) - \int_0^t \left(\dot{p}_c^T (p_c - p_z) - \frac{g}{\eta^2} y_c + r \right) d\tau - E(0) \right)$$





Energy Observer Behaviour



Final displacement

General displacement from original plan

Hybrid Policy

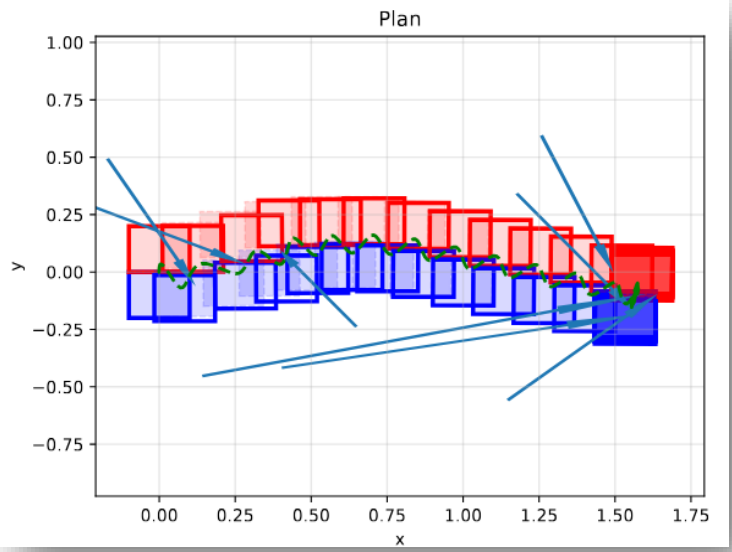
Hybrid Gaussian-Bernoullian policy to decide if acting or not

- Trained to provide predictions both on displacement and on a **binary decision**
- Hybrid action space: continous and discrete

$$p_\theta(s) \in [0, 1]$$

$$b \sim \text{Bernoulli}(p_\theta(s))$$

$$\Delta \sim \mathcal{N}(\mu_\theta(s), \sigma_\theta(s))$$



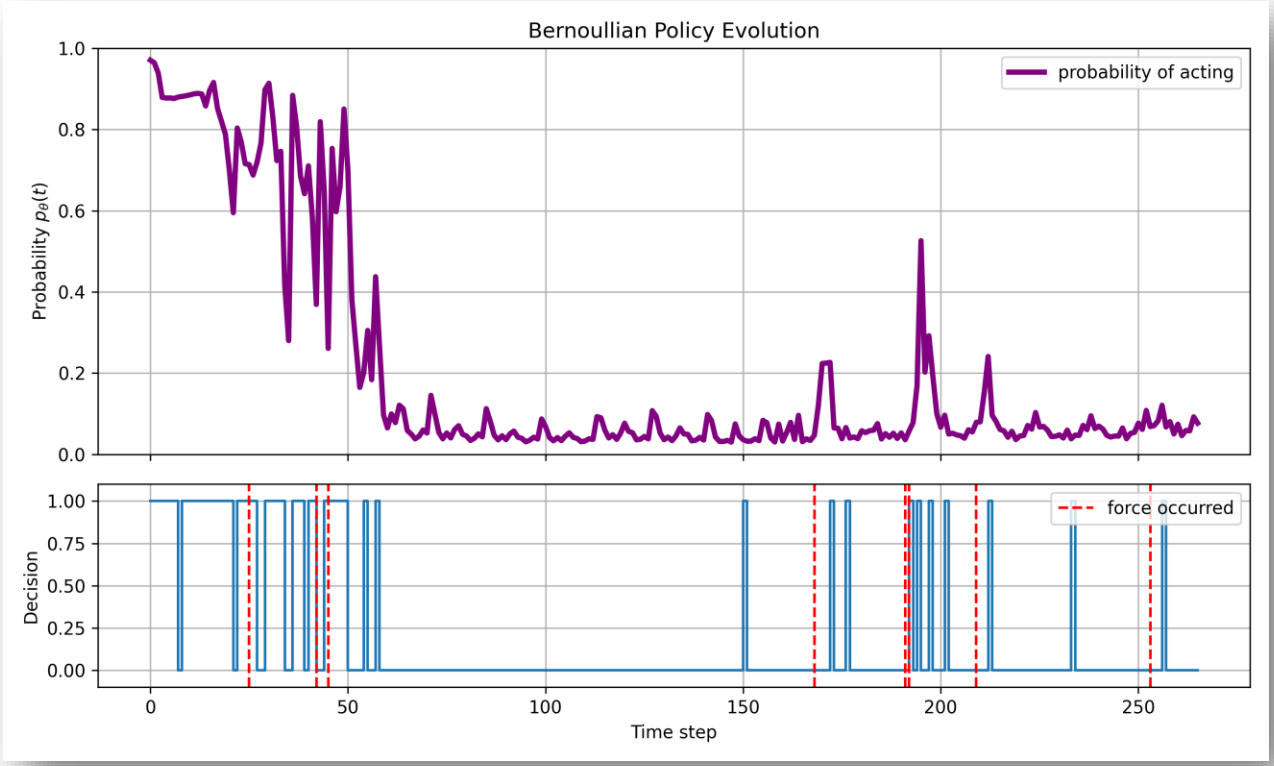
The agent learns to disable corrections to better reproduce the reference

Interventing just in the presence of **external forces** to recover balance

no **ground slope** here

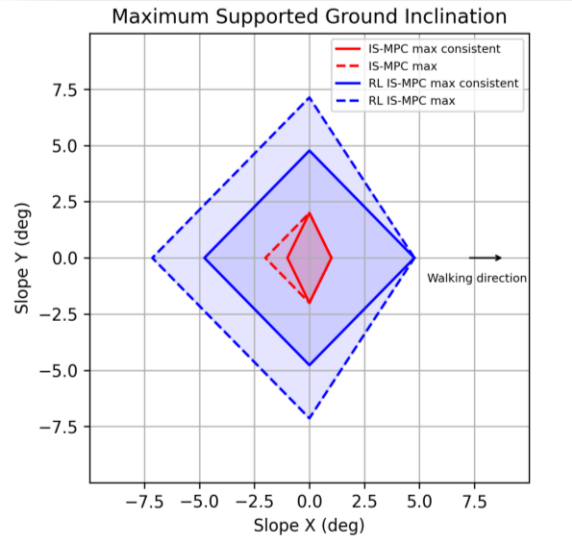
Hybrid Policy

Hybrid Gaussian-Bernoullian policy to decide if acting or not

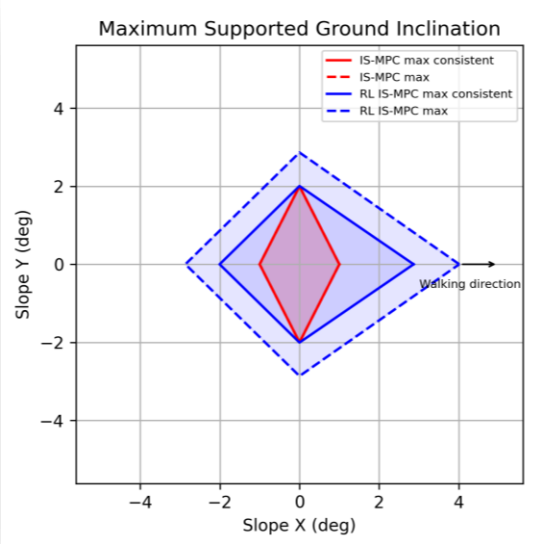


probability of acting increases as a **force** occurs behaves as a **Neural Residual**

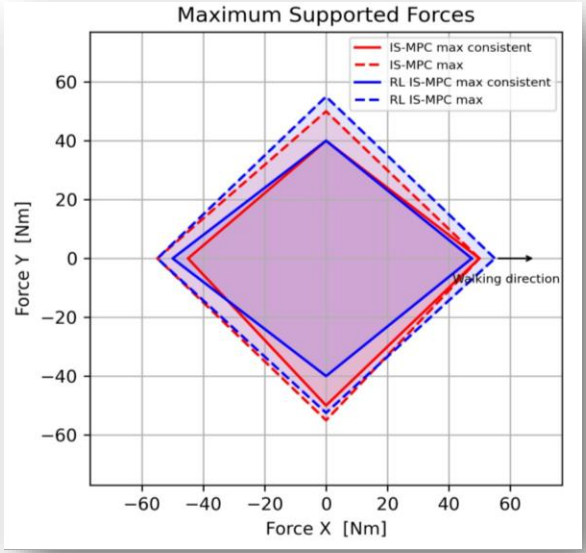
Final comparisons



Slope comparison:
Hybrid Gaussian-Bernoullian policy
VS
pure IS-MPC



Slope comparison:
Approach 3 scaled 0.9
VS
pure IS-MPC



Forces comparison:
Approach 3 scaled 0.9
VS
pure IS-MPC

CONCLUSIONS

Conclusions

Recap of our work

- We augmented a MPC-based control scheme for humanoid locomotion with a Reinforcement-Learning based agent.
- The agent is trained with the **PPO** algorithm, and displaces the footsteps of the plan to **reject disturbances** and keep balance on inclined terrain.
- We explored **three different approaches** to displacement:
  - Only the next footstep: displacement may invalidate the plan. Very good tracking, but poor response to disturbance forces.
  - Whole plan: Saturated displacements are always feasible, but poor tracking.
  - Scaled: best of both worlds.
    - ✓ Better performance when scaling parameter is part of the action space

Conclusions

On generalization

- The agents do not generalize well to new footstep plans:
  - ✓ It's hard to modify the **ZMP/CoM relationship** by just moving footsteps (thus only changing MPC constraints)
  - ✓ IS-MPC assumes **fixed timings** for single and double support phases, in the reference paper they are computed by MPC

- **Saturations** to prevent feet collision lead to better generalization and behaviour in general, by effectively reducing the action space
  - ✓ Less failures -> more regular learning
  - ✓ The agent doesn't know about saturations, but better learns that displacement should be near zero in general, greater only with disturbances

- We also looked to methods for deciding ***when*** to apply the action, to displace footsteps only when strictly necessary, more precise than just saturating actions